## BOOK REVIEWS

W. ODYNIEC AND G. LEWICKI, *Minimal Projections in Banach Spaces*, Springer-Verlag, Lecture Notes in Mathematics, Vol. 1449, 1990, 168 pp.

This is one of the first monographs to treat the topic of minimal projections. A (bounded linear) projection of a Banach space onto a (necessarily closed) subspace is said to be "minimal" if its operator norm is least among all the projections having the same domain and range. Since the family of all projections having a given domain and range is usually an infinite dimensional linear manifold of bounded linear operators, the determination of minimal elements is generally a difficult problem. For example, we do not know the minimal projections from the space of continuous functions on an interval onto the subspace of cubic polynomials. This book opens with a 14-page review of prior work in the field. The four principal chapters are devoted to (i) the unicity of minimal projections, (ii) projections onto subspaces of finite co-dimension, (iii) Kolmogorov criterion for minimal projections, and (iv) isometries of Banach spaces. There is an excellent bibliography of 198 items.

E. W. CHENEY

T. J. RIVLIN, Chebyshev Polynomials: From Approximation Theory to Algebra and Number Theory, Wiley-Interscience, 1990, 249 pp.

This is a much expanded edition of Rivlin's 1974 classic work on the Chebyshev polynomials  $\cos(n \arccos(x))$ . About one-third of the material is new. The first chapter deals with elementary properties including orthogonality. The second chapter is exclusively concerned with extremal problems, while the third chapter discusses expansions in Chebyshev polynomials. The last two chapters cover a miscellany of topics, including iterative behaviour and reducibility. The unifying theme of this book is the Chebyshev polynomials, but by the end of the book the reader has been introduced to virtually all of the central ideas of polynomial approximation theory and many of the techniques. This is not surprising given the central and unique nature of these ubiquitous polynomials. Written in an easy style and well punctuated with exercises, this book provides a wealth of material for the expert while remaining accessible and interesting to a wide audience. This is an attractive book with a long shelf life and is a very welcome addition to any library.

PETER B. BORWEIN

## B. SENDOV, Hausdorff Approximations, Kluwer Academic Publishers, 1990, 364 pp.

Classical evaluation of approximation methods does not always satisfactorily handle discontinuous functions. It often leads to divergence (in the too strong  $L^{\infty}$  norm) or convergence almost everywhere (in the too weak  $L^2$  norm). It is then more accurate to look at graphs of functions, and to measure errors by distances between graphs (*Hausdorff distance*). At this point, it is natural to accept multivalued functions, segment functions, a special brand of interval-valued functions. For instance, the Gibbs effect for Fourier series is accurately taken into account (p. 101). This book, whose Russian edition of 1979 has been praised (MR 80j: 41004), is a remarkable treatise, often shedding new light on usual approximation concepts. The difficult subject of best Hausdorff approximation is covered in 155 pages (Chapters 4 and 5), containing much of the author's own contributions. The chosen topology allows quite interesting results on e-entropy,  $\varepsilon$ -capacity, and widths (Chapter 6). Approximation of plane sets and numerical methods receive a shorter survey (Chapters 7 and 8). This English edition also refers to work done between 1979 and 1990, but P. S. Kenderov (J. Approx. Theory **38** (1983), 221-239) could have been added. Highly recommended!

ALPHONSE P. MAGNUS